E. M. Gol'dfarb and B. I. Sherstov

An approximate analytical solution is presented for the dynamic characteristics of the melting of steel scrap in the oxygen-converter remelting process. A specific calculation is carried out for the melting of a spherical fragment.

The complexity of problems relating to the melting of steel scrap in an iron-carbon melt is due to the interrelated nature of the processes of heat and mass transfer. During the melting process, the temperature, composition, and hydrodynamic parameters of the liquid phase vary, while the pieces of steel scrap are of different shapes and sizes. In the mathematical description of the melting dynamics it is therefore essential to adopt a schematic approach and to make a number of simplifying assumptions. However, even on simplifying the conditions the process can only be described by nonlinear equations. The majority of the computing equations cannot be expressed explicitly with respect to time, and this complicates all the calculations.

The dynamics of scrap melting in an open-hearth furnace were considered in [1, 2]. A distinctive feature of heat and mass transfer in a converter bath (as compared with the open hearth) is the much stronger agitation of the liquid phase and the faster rise in its temperature. Whereas, in an open-hearth furnace, all the scrap melts by virtue of diffusive dissolution over a period in which the temperature of the pig iron is lower than the initial temperature of the steel scrap, in a converter coarse scrap is unable to melt in an analogous period. In the converter-remelting of steel, the pig iron is poured onto the surface of the cold scrap, and (in contrast to open-hearth melting) this compels us to consider the surface freezing and remelting of the melt.

Thus the melting of scrap in the oxygen converter may be divided into the following distinct periods: 1) the freezing of the melt on the surface of the cold material; 2) the melting of the layer so frozen; 3) the diffusive melting (dissolution) of the scrap; 4) the melting of the remaining heated solid material.

The freezing of the melt takes place as a result of the intensive absorption of heat by the cold solid (scrap). The heat evolved in the solidification of the melt is entirely expended in heating the solid material.


Fig. 1. Mode of calculating the freezing of the melt (a) and curve representing the $\mathrm{Bi} \Theta=\mathrm{f}(\mathrm{Fo})$ relationship (b): 1) for a plate; 2) for a cylinder; 3) for a sphere.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 3, pp. 492-498, March, 1970. Original article submitted June 3, 1969.
© 1973 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for $\$ 15.00$.


Fig. 2. Mode of calculating the melting of the frozen melt (a) and curves representing the relationship $(1-\omega) / \mathrm{N}=\mathrm{f}(\mathrm{Fo})(\mathrm{b})$ : 1) for a plate; 2) for a cylinder; 3) for a sphere. Continuous curves: cold solid, Eq. (13); broken curves: heated solid, Eq. (19).

The surface freezing ceases on reaching thermal equilibrium, when the heat fluxes on the inner and outer surfaces of the freezing layer are in balance.

For calculation purposes, we make the assumption that the temperature of the melt is constant and equal to $t_{r}$; the freezing layer is regarded as "thin," at a temperature of $t_{s}$; contact between the freezing layer and the original solid (scrap) is regarded as ideal, and the temperature of the scrap surface is taken as $t_{S}$; we consider that the melt freezes over the whole surface of the solid; the thermophysical constants here and subsequently are regarded as constant.

In order to determine the thickness of the frozen layer and the time of freezing consider the scheme in Fig. Ia.

The thermal flux on the surface of the scrap material (considered in the form of a plate, cylinder, or sphere) when the surface temperature rises instantaneously is equal to [3]

$$
\begin{equation*}
q_{0}=\lambda_{\mathrm{T}}\left(t_{\mathrm{s}}-t_{0}\right) \sum_{n=1}^{\infty} \frac{2}{r_{0}} \exp \left(-\mu_{n}^{2} \frac{a_{\mathrm{T}} \tau}{r_{0}^{2}}\right) . \tag{1}
\end{equation*}
$$

The amount of heat arriving on unit surface area in a time $\tau$ equals [3]

$$
\begin{equation*}
Q=\lambda_{\mathrm{T}}\left(t_{s}-t_{0}\right) \sum_{n=1}^{\infty} \frac{2 r_{0}}{\mu_{n}^{2} a_{\mathrm{T}}}\left[1-\exp \left(-\mu_{n}^{2} \frac{a_{\mathrm{T}} \tau}{r_{0}^{2}}\right)\right] . \tag{2}
\end{equation*}
$$

Let us set up the thermal balance equation corresponding to a period $d \tau$ for the solid and the layer of frozen melt

$$
\begin{equation*}
\rho_{r}\left[c_{r}\left(t_{r}-t_{s}\right)+x\right] d V=f_{0} q_{0} d \tau \tag{3}
\end{equation*}
$$

The ratio $\mathrm{V} / \mathrm{f}_{0}$ may be expressed as follows:

$$
\begin{equation*}
\frac{V}{f_{0}}=\frac{r_{0}\left(\omega_{i}^{2 v+2}-1\right)}{(2 v+2)}, \tag{4}
\end{equation*}
$$

where $\omega_{1}=1+\mathrm{s} / \mathrm{r}_{0}$.
After integrating (3) and allowing for (4) we have

$$
\begin{equation*}
\omega_{1}^{2 v+2}-1=\frac{(2 v+2) Q}{r_{0} \rho_{r}\left[c_{r}\left(t_{r}-t_{s}\right)+x\right]}, \tag{5}
\end{equation*}
$$

where $Q$ is determined from the condition of thermal equilibrium. For this purpose we must solve the following equation for $\tau$

$$
\begin{equation*}
\alpha\left(t_{r}-t_{\mathrm{s}}\right)=\lambda_{\mathrm{T}}\left(t_{\mathrm{s}}-t_{0}\right) \sum_{n=1}^{\infty} \frac{2}{r_{0}} \exp \left(-\mu_{n}^{2} \frac{a_{\mathrm{T}} \tau}{r_{0}^{2}}\right) \tag{6}
\end{equation*}
$$



Fig. 3. Mode of calculating the dissolution of scrap (a) and curve representing the relation $\left(\mathrm{C}_{\mathrm{s}}-\mathrm{C}_{0}\right) /\left(\mathrm{C}_{\mathrm{p}}-\mathrm{C}_{\mathrm{S}}\right)$ $=f(\xi)$ (b).

The solution (6) is shown graphically in Fig. 1b in the form of the function

$$
\operatorname{Bi} \Theta=f(\mathrm{Fo})
$$

where

$$
\mathrm{Bi}=\frac{\alpha r_{0}}{\lambda_{\mathrm{T}}} ; \quad \Theta=\frac{t_{r}-t_{\mathrm{s}}}{t_{\mathrm{s}}-t_{0}} ; \quad \mathrm{FO}=\frac{a_{\mathrm{T}} \tau}{r_{0}^{2}} .
$$

A solution was presented in [9] for the freezing and melting of a crust of melt; this gave results approaching the exact solution of [10] for large values of $\lambda_{T} / \lambda_{r}, c_{T} / c_{r}$. In deriving the computing equations the scrap material was regarded as "thin," so that its temperature was independent of the coordinate $x$. For the "bulk" materials which are of major interest in the melting of converter scrap, the foregoing calculation [9] introduces serious errors. Calculations based on Eq. (5), however, in contrast to those of [9], give more accurate results for larger values of $\lambda_{r} / \lambda_{T}$.

The melting of the frozen layer starts after thermal equilibrium has been achieved. In order to derive the equation of the melting process we consider the following scheme (Fig. 2a). We derive an equation for the continuous removal of melt from the surface of the solid material ( $\left.\mathrm{R}_{0}=\mathrm{r}_{0}+\mathrm{s}_{\mathrm{H}}\right)$. The thermal flux is expended in melting an elementary layer and heating the rest of the solid. The thermophysical constants are taken as constant, and averaged for the frozen melt and scrap. Let us set up the thermal balance equation over a period $\mathrm{d} \tau$ for the melting solid

$$
\begin{equation*}
q_{r} f_{1} d \tau=f_{1} x \rho d s-d\left[\rho c V_{1}\left(t_{s}-t_{\mathrm{av}}\right)\right] \tag{7}
\end{equation*}
$$

After introducing the values of $\mathrm{tav}_{\mathrm{av}}$ and $\mathrm{V}_{1}$ we obtain [3]

$$
\begin{equation*}
q_{r} d \tau=x \rho d s-\frac{d\left[q_{1}\left(R_{0}-s\right)^{2 v+3}\right]}{a(2 v+2)(2 v+4)\left(R_{0}-s\right)^{2 v+1}} . \tag{8}
\end{equation*}
$$

The solution (8) was obtained in [3] by introducing the concept of a nominal specific heat.
In the present instance, in order to determine $q_{1}$, we take $s=0$. After transforming (8) we obtain

$$
\begin{equation*}
q_{r} d \tau-x \rho d s=-\frac{R_{0}^{2} d q_{1}}{a(2 v+2)(2 v+4)} \tag{9}
\end{equation*}
$$

Equation (7) may be written in a different way

$$
\begin{equation*}
q_{r} d \tau-x \rho d s=q_{1} d \tau \tag{10}
\end{equation*}
$$

Comparing (9) and (10) we obtain

$$
\begin{equation*}
q_{1} d \tau=-\frac{R_{0}^{2} d q_{1}}{a(2 v+2)(2 v+4)} \tag{11}
\end{equation*}
$$

Integrating (11) and considering that for $\tau=0 q_{1}=q_{r}$, we obtain

$$
\begin{equation*}
q_{\mathrm{t}}=q_{r} \exp \left[-\frac{(2 v+2)(2 v+4) a \tau}{R_{0}^{2}}\right] \tag{12}
\end{equation*}
$$

After substituting (12) into (9) and integrating, we obtain an expression which may be used for determining the time of melting of the frozen melt and the motion of the melting boundary

$$
\begin{equation*}
(2 v+2) \mathrm{Fo}_{\mathrm{o}}=\frac{1-\omega}{2 N}+\frac{1}{2 v+4}[1-\exp (-A \mathrm{Fo})], \tag{13}
\end{equation*}
$$

and

$$
\omega=1-\frac{s}{R_{0}} ; A=(2 v+2)(2 v+4) ; N=\frac{c q_{r} R_{0}}{2(2 v+2) r \lambda} .
$$

The solution of (13) is given in Fig. 2b.
In order to estimate the accuracy of the solution, we made comparative calculations based on Eq. (13) and the nomograms of [4] for various $\nu$ and $N$. The calculations showed that Eq. (13) was reasonably accurate.

The diffusive melting of the solid involves both heat and mass transfer. The process taking place at the lowest velocity at any specified instant of time will constitute the limiting factor.

Let us consider the dynamics of scrap dissolution starting from the diffusion of carbon in the solid material. The character of the carburization process will be determined by the quantity $\mathrm{Bi}_{\mathrm{d}}=\beta \mathbf{r}_{0} / \mathrm{D}$. Under the conditions encountered in the oxygen converter, in which the melt suffers intensive agitation, $\mathrm{Bi}_{\mathrm{d}} \gg 1$. Hence a carbon concentration corresponding to the liquidus line should be established instantaneously at the surface of the solid. In order to determine the motion of the dissolution boundary, we use the solution derived in [3] for a plate (Fig. 3a):

$$
\begin{gather*}
\frac{\partial C(x, \tau)}{\partial \tau}=D \frac{\partial^{2} C(x, \tau)}{\partial x^{2}}, s \leqslant x \leqslant \infty ; \\
C(x, 0)=C(s, \tau), C(\infty, 0)=C_{0} ;  \tag{14}\\
D \frac{\partial C(s, \tau)}{\partial x}=\left(C_{p}-C_{s}\right) \frac{\partial \dot{s}}{\partial \tau} ;
\end{gather*}
$$

$r_{0}$ being regarded as infinite. According to [3] the solution of Eq. (14) will have the form

$$
\begin{equation*}
\frac{C(u)-C_{p}}{C_{s}-C_{p}}=\frac{\operatorname{erfc} u}{\operatorname{erfc} \xi} \tag{15}
\end{equation*}
$$

where

$$
u=\frac{x}{2 \vartheta \overline{D \tau}} ; \xi=\frac{s}{2 \gamma \overline{D \tau}} ;
$$

$\xi$ is determined from the equation

$$
\begin{equation*}
\frac{C_{s}-C_{0}}{C_{p}-C_{s}}=\sqrt{\pi} \xi \exp \xi^{2} \operatorname{erfc} \xi \tag{16}
\end{equation*}
$$

For approximate calculations with $\left(\mathrm{C}_{s}-\mathrm{C}_{0}\right) /\left(\mathrm{C}_{\mathrm{p}}-\mathrm{C}_{\mathrm{s}}\right) \geq 0.8$, instead of (16) we may use the equation

$$
\begin{equation*}
\frac{C_{s}-C_{p}}{C_{p}-C_{s}}=\left(1-\frac{1}{2 \xi^{2}}\right) \tag{1.7}
\end{equation*}
$$

The motion of the dissolution boundary is determined by the equation

$$
\begin{equation*}
s=2 \xi \sqrt{D \tau} \tag{18}
\end{equation*}
$$

The diffusion dissolution of the scrap lasts approximately $10-15 \mathrm{~min}$ after the blast begins. Calculations show that in this period only a very thin layer is able to dissolve, and the use of Eq. (18) for a cylinder or sphere introduces little error into the solution.

The melting of the heated solid starts from the instant at which the temperature of the melt exceeds the melting point of the original scrap. In deriving the corresponding equation, we may use the principles employed earlier in calculating the melting of the frozen melt. However, we must remember that in this case the solid has already been heated, and the thermal flux will be expended solely in melting it. The dynamic equation will take the form


Fig. 4. Dynamics of the melting of a sphere: I) freezing of the melt; II) remelting of the frozen melt; III) dissolution of the solid; IV) melting of the remaining heated part of the solid; $t$, min.

$$
\begin{equation*}
(2 v+2) \mathrm{Fo}=\frac{1-\omega}{2 N} . \tag{19}
\end{equation*}
$$

The solution of (19) is presented in Fig. 2b.
A fundamental difficulty in calculating scrap-melting dynamics by the method proposed is that of determining the heat-transfer coefficient. There are as yet no reliable data for the conditions of the oxygen converter, such as might be used for these calculations. Results such as those of [1, 2], based on a study of the melting of scrap in an open-hearth furnace, cannot be mechanically transferred to converter melting, since the dependence of the coefficient $\alpha$ on the intensity of agitation of the liquid phase is clearly not linear. Without experimental data we can only make a rough guess as to the value of this coefficient. To a first approximation the average $\alpha$ for the period of intensive decarburization may be determined from the thermal balance of the water cooling the tuyere. For this purpose we require to know the depth of immersion of the tuyere in the melt as well as its structural dimensions and the temperature and rate of flow of the water.

Calculations based on published data and actual practice show that for this period $\alpha$ lies in the range $2 \cdot 10^{4}-3 \cdot 10^{4} \mathrm{~W} / \mathrm{m}^{2} \cdot$ deg. For the period of freezing and remelting of the melt, $\alpha$ may be taken as having the average value derived from [1, 7]. Figure 4 shows the melting curve of a sphere 200 mm in diameter. Specific data for the melt and the scrap in the calculation were taken from [5,6] and experimental results. The diffusion coefficient of carbon in steel scrap at $\mathrm{T}=1700^{\circ} \mathrm{K}$ was determined [8] from

$$
D=\left\{0,07+0.06[C] \exp \left(-\frac{32000}{1.99 T}\right)\right\}
$$

The ratio $\left.\left(C_{S}-C_{0}\right) / C_{p}-C_{S}\right)$ was taken as equal to 0.8 . The value of the thermal flux $q_{r}$ for the period of intensive melting (period IV) was taken as $1.25 \cdot 10^{6} \mathrm{~W} / \mathrm{m}^{2}$, which corresponds to $\alpha=2.5 \cdot 10^{4} \mathrm{~W} / \mathrm{m}^{2} \cdot$ deg and $\mathrm{t}_{\mathrm{r}}-\mathrm{t}_{\mathrm{S}}=50^{\circ} \mathrm{K}$.

The results of the calculations showed that: the time of freezing and remelting of the melt respectively equalled 30 and 105 sec ; the thickness of the frozen layer was 20 mm ; during the diffusive-dissolution period ( 12 min ), the diameter of the sphere diminished by 5 mm ; the remaining heated part of the sphere melted in 4 min . The total time for the melting of the sphere was 18 min . If we remember that the pouring of the pig iron lasted some 2 min , we see that the sphere melted 16 min after the blast started.

We see from the calculations that, on using coarse scrap, the cooling of the bath during melting periods I, II, and III may be mainly ascribed to the heat required for heating the scrap.

The use of "light-weight" scrap with a highly developed surface causes intensive cooling of the bath at the onset of melting by virtue of its heating and dissolution; this is undesirable for technological reasons, since the process of slag formation is retarded at low temperatures.

The computing method presented here provides a general picture of the melting of cold scrap in a converter bath.

## NOTATION

```
t
t
\alpha and \beta are the heat- and mass-transfer coefficients;
x is the latent heat of melting;
a is the thermal diffusivity;
\lambda is the thermal conductivity;
\rho is the density;
c is the specific heat;
q}\mp@subsup{|}{0}{}\mathrm{ and qr are the specific thermal fluxes arriving at the surface of the immersed solid during the freez-
    ing and remelting of the melt;
```

| $\mathrm{q}_{1}$ | is the specific thermal flux used in heating a layer ( $\mathrm{r}_{0}-s$ ) thick; |
| :---: | :---: |
| $\mathrm{C}_{\mathrm{r}}$ | is the carbon concentration in pig iron; |
| $\mathrm{C}_{0}$ | is the initial carbon concentration in the scrap; |
| $\mathrm{C}_{\mathrm{p}}$ and $\mathrm{C}_{S}$ | are the carbon concentrations on the liquidus and solidus corresponding to the specified metal temperature; |
| $\mathrm{s}_{\mathrm{H}}$ | is the maximum thickness of the frozen layer of melt on the scrap; |
| s | is the current coordinate of the phase-transformation boundary; |
| $\mathrm{r}_{0}$ | is the initial size of the solid; |
| $\nu$ | is the form factor of the solid ( $\nu=1 / 2,0$, and $1 / 2$ for a plate, cylinder, and sphere); |
| $\mathrm{V}_{1}$ and $\mathrm{f}_{1}$ | are the volume and area of the solid; |
| $\tau$ | is the time; |
| $\mu_{\mathrm{n}}$ | are the roots of the equation governing the heating of the solid (plate, cylinder, or sphere) when the temperature of its surface rises instantaneously; |
| D | is the diffusion coefficient of carbon in the scrap. |

## LITERATURE CITED

1. I. B. Belov, Izv. Akad. Nauk SSSR, Metally, No. 6 (1966).
2. E. A. Kapustin, Izv. VUZ, Chernaya Met., No. 7 (1965).
3. É. M. Gol'dfarb, Heat Technology of Metallurgical Processes [in Russian], Metallurgiya (1967).
4. N. P. Svinolobov and I. D. Semikin, Izv. VUZ, Chernaya Met., No. 1 (1963).
5. Steel-Melting Industry, Handbook, Vols. I and II [in Russian], Metallurgiya (1964).
6. S. G. Afanas'ev, Short Handbook for Converter Workers [in Russian], Metallurgiya (1967).
7. A. S. Nevskii and A. I. Malysheva, in: Transactions of the All-Union Scientific--Research Institute of Metallurgy[in Russian], No. 12 (1965).
8. M. A. Krishtal, Diffusion Processes in Iron Alloys [in Russian], Metallurgiya (1963).
9. M. Ya. Antimirov, Latvian Mathematical Annual [in Russian], Zinatne (1966).
10. A. A. Buikis, L. I. Rubinshtein, and A. B. Skroman, Latvian Mathematical Annual [in Russian], Zinatne (1966).
